CDF public note 11084

Measurement of the top quark mass with *in situ* jet energy scale calibration in the all-hadronic channel using the Template Method with 9.3 fb^{-1}

The CDF Collaboration
URL http://www-cdf.fnal.gov
(Dated: March 28, 2014)

We present here the measurement of the top quark mass with simultaneous (*in situ*) calibration of the Jet Energy Scale (JES), by the Template Method in the all-hadronic channel, i.e. where both W's decay into $q\bar{q}'$ pairs. The measurement discussed here is performed using about $9.3\,\mathrm{fb}^{-1}$ of $p\bar{p}$ collisions collected with a multijet trigger at $\sqrt{s} = 1.96$ TeV with the Collider Detector at Fermilab (CDF). The method relies on the comparison, for events selected by a Neural Network, of the reconstructed top quark and W boson masses distributions in the data to expectation from signal Monte Carlo and data-driven background events, to extract the top mass and the JES through an unbinned likelihood technique. The measurement gives a top quark mass $\mathrm{M_{top}} = 175.07\ \pm 1.19\,(stat)\ \pm 1.56\,(syst)\,\mathrm{GeV}/c^2$.

Preliminary Results for Winter 2014 Conferences

I. INTRODUCTION

At the Fermilab TEVATRON, top quarks are mainly pair produced in $p\bar{p}$ collisions via $q\bar{q}$ annihilation (85%) and gluon-gluon fusion (15%). According to the Standard Model, the top quarks decay into W bosons and b quarks with a branching ratio (BR) about equal to 1. In this analysis we search for events in which both W bosons decay into quark pairs, leading to an all-hadronic final state. This channel has the advantage of the largest BR, about 46%, and of the fully reconstructed kinematics. The major downside is the huge background from QCD multijet production which dominates the signal by three orders of magnitude even after the application of the specific top multijet trigger. A sophisticated event selection based on kinematical and topological variables, followed by the request of identified b-jets, is thus needed in order to further improve the signal to background ratio (S/B).

We present here a measurement of the top quark mass performed using about $9.3\,\mathrm{fb}^{-1}$ of data. Distributions (templates) of variables sensitive to the main observables we want to measure, i.e. the top quark mass ($\mathrm{M_{top}}$) and the jet energy scale (JES) to be applied to jets in simulated events to match the data, are built and used to discriminate the possible values of these variables. At the same time, the differences between signal and background distributions allow to estimate the respective average contributions to the observed events, so that the measurement can be obtained by maximizing a likelihood fit of the signal and background templates to the data. As we use templates to measure, above all, two quantities simultaneously, i.e. $\mathrm{M_{top}}$ and JES, the technique is referred to as TMT2D (Top Mass Templates 2-Dimensional measurement). The reliability of the method, its expected performance and the main sources of systematic uncertainties have been evaluated by large sets of simulated experiments (*pseudo-experiments*) before the actual measurement on the data.

II. EVENT SELECTION

All data and simulated Monte Carlo events, previously selected by a multijet trigger, have to pass some prerequisites which require a well centered primary vertex and no lepton with high transverse momentum (p_T) identified in the event. The events satisfying this first selection are then required to have a number of detected "tight" jets (i.e. jets with $E_T \geq 15$ GeV, $|\eta| \leq 2.0$) between 6 and 8 with a minimum distance between each pair of jets in the (η, ϕ) plane (ΔR_{min}) larger than 0.5, and no significant missing transverse energy. A number of kinematic variables are then reconstructed using tight jets and serve as inputs to a neural network with 13 input variables, one hidden layer and one output layer. As described in [1], the 13 inputs include both variables depending on energy and direction of jets, and also on their shape. The latter are very effective in distinguishing jets produced by light flavor quark (present in signal events) from the wider jets initiated by gluons, in principle typical of background events only.

Events are selected if the output value from the neural network, NN_{out}, exceeds a given threshold. Finally we require the presence of jets tagged as b-jets among the six leading jets, and subdivide our sample in events with exactly one tagged jet (1-tag sample) and two or three tagged jets ($\geq$ 2-tags sample). A jet is tagged if some of its tracks form a secondary vertex significantly displaced from the interaction point. Different values of the NN_{out} threshold are chosen for the two categories of tagged events, in such a way to maximize the statistical significance of the mass measurement, as described in section VI.

Signal Monte Carlo events have been generated with POWHEG [2] interfaced to PYTHIA [3] 6.4.18 for the parton shower, with values of input $\mathrm{M_{top}}$ in the range between 167.5 and $177.5\,\mathrm{GeV}/c^2$, in GeV/c^2 steps. The event selection is repeated changing the value of the JES from $-2\,\sigma_{\mathrm{JES}}$ to $+2\,\sigma_{\mathrm{JES}}$, in steps of $0.5\,\sigma_{\mathrm{JES}}$, with respect to its central value as measured in [4], where σ_{JES} is the uncertainty on that value itself. In the following we then evaluate the JES in terms of its displacement, ΔJES, from the nominal value (corresponding therefore to $\Delta\mathrm{JES} = 0$) and using σ_{JES} as the unit.

III. BACKGROUND MODELING

The background consists mainly of QCD production of light and heavy flavor quarks. Its modeling and estimate are data-driven and based on a tag rate parametrization derived in a sample of events with exactly 5 jets and therefore dominated by the background. The probability to tag a jet is parametrized according to the jet-E_T, jet track multiplicity, and number of well-defined vertices in the event, and can then be applied to taggable jets (i.e. jets accepted by the b-tagging algorithm) identified in events selected by the kinematic requirements, to evaluate the inclusive number of tagged jets originating from background events. However, a direct exploitation of the tag rate matrix to predict the number of background events with a given number of tags would give incorrect numbers because the matrix, by construction, refers to an inclusive tagging probability and does not consider that in QCD background real heavy flavour quarks come in pairs and have therefore an enhanced double-tagging probability, so that the probability to tag a pair of jets in the same event is not simply equal to the product of the tag rates of single jets.

To account for this we introduce correction factors to obtain a better estimate for the number and distributions of 1-tag and $\geq$ 2-tags events due to the background. These factors are derived in a control sample dominated by the background (events with 6-8 jets and $NN_{out} \leq 0.50$, where the signal contribution is still negligible) and represent average corrections to the probability for a possible "tag configuration", that is for the assumption that given taggable jets in an event in the pretag sample are the only tagged jets in the same event after b-tagging.

The data-driven background prediction must be performed starting from events in the pretag sample, but, as this contains also events from $t\bar{t}$ signal, the raw prediction must be corrected to take these into account.

In the analysis presented here no *a-priori* prediction of the number of expected background events is actually considered. This means that in the likelihood fit each parameter concerning the background normalization is free and only shapes of background templates are used.

IV. BUILDING TEMPLATES

The $t\bar{t}$ events under study in this work are characterized by the nominal presence of 6 quarks in the final states, two of which are b-quarks. Therefore, the signal signature would ideally consist of 6 reconstructed jets in the detector, with some being tagged as b-jets. We want to fully reconstruct the kinematics of events passing the kinematical selection, partially described in section II, and exploit the presence of the W and top quark to constrain the event topology. In order to do so we consider only the 6 leading (in E_T) jets in the event to limit the number of ways in which we can combine the jets to reconstruct the events. There are 90 possible different jet-to-parton associations with two jet doublets giving a W and two jet triplets giving the top quarks. Since we consider only events with tagged jets, we further reduce the number of permutations by requiring the association of the b-tagged jets to a b quark; we are therefore left with 30 possible parton-jet assignment in 1-tag sample, and 6 or 18 in the $\geq$ 2-tags sample[13].

A. m_t^{rec} templates

We reconstruct the kinematic of the event by a fit based on the following χ^2-like quantity:

$$\chi_t^2 = \frac{\left(m_{jj}^{(1)} - M_W\right)^2}{\Gamma_W^2} + \frac{\left(m_{jj}^{(2)} - M_W\right)^2}{\Gamma_W^2} + \frac{\left(m_{jjb}^{(1)} - m_t^{rec}\right)^2}{\Gamma_t^2} + \frac{\left(m_{jjb}^{(2)} - m_t^{rec}\right)^2}{\Gamma_t^2} + \sum_{i=1}^{6} \frac{\left(p_{T,i}^{fit} - p_{T,i}^{meas}\right)^2}{\sigma_i^2}$$

where $m_{jj}^{(1,2)}$ are the invariant masses of the dijet systems assigned to light flavor quarks, $m_{jjb}^{(1,2)}$ are the invariant masses of the trijet systems including one b-quark, $M_W = 80.4\,\mathrm{GeV}/c^2$ and $\Gamma_W = 2.1\,\mathrm{GeV}/c^2$ are the measured mass and natural width of the W boson [5], and $\Gamma_t = 1.5\,\mathrm{GeV}/c^2$, is the assumed natural width of the top quark. The measured jet transverse momenta can vary, but are constrained to the measured value, $p_{T,i}^{meas}$, within their known resolution, σ_i.

For each permutation of the jet-to-parton assignments in the event, χ_t^2 is minimized with respect to 7 free parameters, i.e. the reconstructed top quark mass, m_t^{rec}, and the 6 jets transverse momenta $p_{T,i}^{fit}$ and the combination which gives the lowest value for the χ_t^2 minimum is selected. The m_t^{rec} value corresponding to this permutation enters an invariant mass distribution, i.e. the template which will serve as a reference for the $\mathrm{M_{top}}$ measurement. This procedure is repeated on selected signal Monte Carlo events with all the different input values of $\mathrm{M_{top}}$ and ΔJES and, to parametrize a continuous dependence of the m_t^{rec} templates on these parameters, we perform a fit of the distributions to functional forms which vary smoothly with respect to them. So, we obtain probability density functions (p.d.f.'s) which we will use to form an unbinned likelihood for the final measurement. The signal p.d.f., $P_s^{m_t^{rec}}(m_t|\mathrm{M_{top}}, \Delta\mathrm{JES})$, represents the probability to obtain a value m_t for m_t^{rec}, given a true top quark mass $\mathrm{M_{top}}$ and a true value ΔJES of the displacement of the jet energy scale, in a $t\bar{t}$ event.

B. m_W^{rec} templates

Reconstructing the mass of W bosons by dijet systems represents a possibility to obtain a variable in principle insensitive to $\mathrm{M_{top}}$ which allows, therefore, a determination of JES not dependent on $\mathrm{M_{top}}$ itself.

To build the m_W^{rec} templates we use the same procedure and χ^2 expression considered for m_t^{rec} templates, but now also the W mass is left as a free parameter in the fit (i.e. M_W becomes m_W^{rec}). The χ^2-like function obtained this way is denoted χ^2_W. Again, for each event, the value of m_W^{rec} corresponding to the permutation of the jet-to-parton assignments with the lowest minimized χ^2_W enters the template, and this procedure is repeated on selected signal Monte Carlo events with all the different input values of $\mathrm{M_{top}}$ and ΔJES. Like for m_t^{rec}, also the m_W^{rec} templates need to be parametrized by functions depending continuously on $\mathrm{M_{top}}$ and ΔJES. The m_W^{rec} p.d.f., $P_s^{m_W^{rec}}(m_W|\mathrm{M_{top}}, \Delta\mathrm{JES})$, represents the probability to obtain a value m_W for m_W^{rec}, given true inputs $\mathrm{M_{top}}$ and ΔJES, in a $t\bar{t}$ event.

C. Background m_t^{rec} and m_W^{rec} templates

Given our model, we must build the background m_t^{rec} and m_W^{rec} templates considering events and taggable jets (instead of tagged ones) in the pretag sample. In particular all the possible combinations where 1, 2, or 3 *taggable jets* among the 6 leading jets are *assumed* as tagged must be considered, and, for each combination, the same procedures described in sections IV A and IV B must then be repeated to extract corresponding values of m_t^{rec} and m_W^{rec}. These values then enter the templates weighted by the *corrected* probability (see section III) that the jets assumed as tagged in the combination are effectively the tagged ones in the event after b-tagging. Corrections for the presence of signal events in the pretag sample must be taken into account, and the corresponding contribution to the shape subtracted. No dependence on $\mathrm{M_{top}}$ and JES is considered for the background templates, but effects of differences due to corrections performed by signal events corresponding to different values of these variables are taken into account by the calibration procedure (section VIII B). The background p.d.f.'s, $P_b^{m_t^{rec}}(m_t)$ and $P_b^{m_W^{rec}}(m_W)$, represent the probabilities to obtain values m_t for m_t^{rec} and m_W for m_W^{rec} respectively, in a background event.

V. BACKGROUND VALIDATION

In order to check how properly our modeling describes the background, we consider events in control regions defined by the NN_{out} value, in ranges where the signal presence after tagging is still very low. In these regions the templates, i.e. the distributions which are essential to our measurement, are reconstructed by the procedure described in the previous sections both for the signal and the background. As the final selections of the data samples include cuts on the NN_{out} value and on the χ^2 functions of the fits used to build the m_W^{rec} and m_t^{rec} templates (denoted in the following by χ^2_W and χ^2_t respectively), as it will be described in section VI, also these distributions are really important. Obviously, as it concerns the background, they must be evaluated by the same procedure of weighting each assumed possible configuration with 1, 2 or 3 tagged jets described in section IV C for the templates.

The agreement between expected and observed distributions is rather good in all the control regions, with some small exception. This confirms the general reliability of the background model, even if systematic uncertainties concerning the shape of background distributions have to be considered. In Fig. 1 the output of the neural network over the whole range of values $NN_{out} > 0.5$ is shown, while Figures 2 and 3 show distributions of χ^2_W, χ^2_t, m_W^{rec} and m_t^{rec} in one of the control regions both for 1-tag and $\geq$ 2-tags events, where the sum of signal and background is compared to the same distributions reconstructed in the data. In these plots the signal distributions corresponding to $\mathrm{M_{top}} = 172.5\,\mathrm{GeV}/c^2$ and $\Delta\mathrm{JES} = 0$ have been normalized assuming $\sigma_{t\bar{t}} = 7.45\,\mathrm{pb}$ [6], while the amount of background events corresponds to the difference beteween the number of observed events and the expected signal.

VI. EVENTS SAMPLES

The requirements defining the event samples used for this analysis have not been changed with respect the last measurement in the same decay channel [7], where they were set by pseudoexperiments in order to minimize the uncertainty expected for $\mathrm{M_{top}}$ from the likelihood fit.

Two different samples of events, denoted by S_{JES} and $S_{\mathrm{M_{top}}}$, are defined and used to build the m_W^{rec} and m_t^{rec} templates, respectively. The set S_{JES} is selected by requirements on NN_{out} and χ^2_W, while $S_{\mathrm{M_{top}}}$ by a *further* requirement on χ^2_t, so that $S_{\mathrm{M_{top}}} \subseteq S_{\mathrm{JES}}$. As S_{JES} is somehow used to calibrate the JES, while $S_{\mathrm{M_{top}}}$ is more strictly related to the top quark mass measurement, we also refer to S_{JES} and $S_{\mathrm{M_{top}}}$ as "JES-sample" and "$\mathrm{M_{top}}$-sample" respectively.

We therefore finally set, besides the prerequisites described in section II:

- 1-tag events:
  - S_{JES} sample: $NN_{out} \geq 0.97$, $\chi^2_W \leq 2$ and 1 tagged jet;

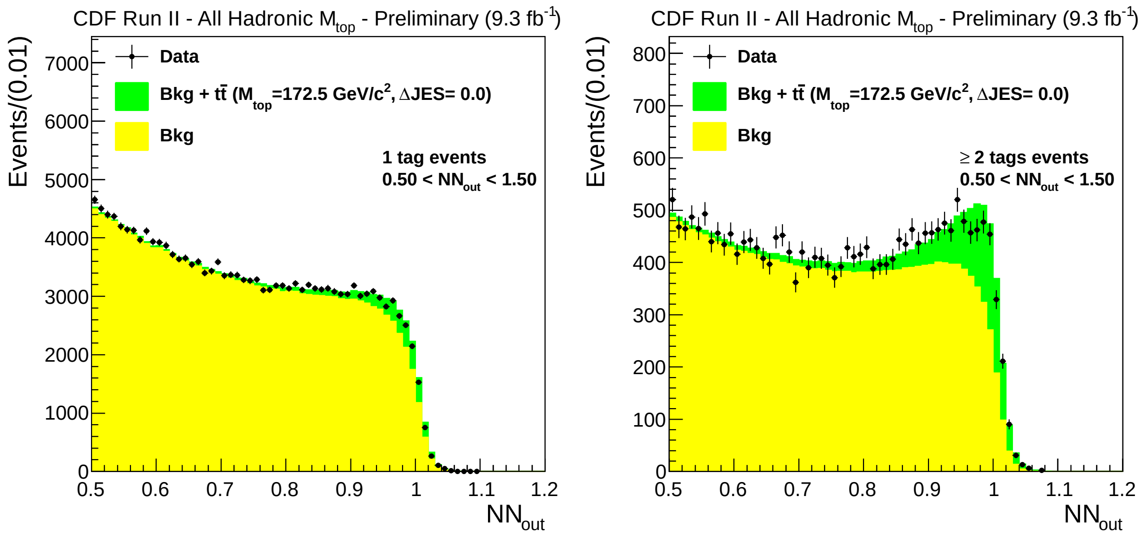

FIG. 1: Distributions of the output from the Neural Net, NN_{out}, for 1-tag events, left plot, and $\geq$ 2-tags events, right plot, are shown in the whole region defined by $NN_{out} > 0.5$. Along with the data are plotted the corrected expected background and the signal contribution. We see that the agreement is generally good.

– $S_{\mathrm{M_{top}}}$ sample: $NN_{out} \geq 0.97$, $\chi^2_W \leq 2$, $\chi^2_t \leq 3$ and 1 tagged jet;

- $\geq$ 2-tags events:

  – S_{JES} sample: $NN_{out} \geq 0.94$, $\chi^2_W \leq 3$ and 2 or 3 tagged jets;

  – $S_{M_{top}}$ sample: $NN_{out} \geq 0.94$, $\chi^2_W \leq 3$, $\chi^2_t \leq 4$ and 2 or 3 tagged jets;

as the requirements and the samples to be used in our analysis.

For $t\bar{t}$ events corresponding to $\mathrm{M_{top}} = 172.5\,\mathrm{GeV}/c^2$ and $\Delta\mathrm{JES} = 0$, the values of the efficiencies of the JES-sample selections are 2.7% and 1.1% for 1-tag and $\geq$ 2-tags respectively, while for the corresponding $\mathrm{M_{top}}$-samples we obtain 1.8% and 0.7%. For the same $\mathrm{M_{top}}$ and ΔJES, the fraction of events of the JES-sample selected by the requirements on χ^2_t only, and therefore belonging to the $\mathrm{M_{top}}$-sample, are 67.3% and 65.8%, as can be inferred by the ratios of the absolute efficencies. These latter acceptances will be denoted by $\mathcal{A}_s$ in the following and their values generally depend on $\mathrm{M_{top}}$ and ΔJES.

We can now also evaluate the observed number of events in our final samples, as well as the expected amounts of signal and background. They are summarized in Tab. I. As outlined above, in this analysis no numerical prediction for the number of background events is used, so the difference between the number of events observed in the data and the expected contribution of selected $t\bar{t}$ events is used. The quoted uncertainties on the the latter is due to the statistics of Monte Carlo samples, the uncertainty on the integrated luminosity of the data, and the theoretical uncertainty on the production cross section [6].

A. Parametrizations

Having defined the requirements for this analysis, we can proceed to build the signal and background templates from events in the selected samples and, for the signal, to parametrize their dependence on $\mathrm{M_{top}}$ and ΔJES into smooth probability density functions. The method have been already described in section IV. Figures 4 and 5 show the fitted p.d.f.'s superimposed to the m_t^{rec} and m_W^{rec} signal templates respectively for different $\mathrm{M_{top}}$ and ΔJES values.

The background m_t^{rec} and m_W^{rec} templates and the corresponding fitted parametrized p.d.f.'s for the signal region are shown in Figure 6 both for 1-tag and $\geq$ 2-tags events.

For signal events, also the acceptances $\mathcal{A}_s$ defined in section VI depend on $\mathrm{M_{top}}$ and ΔJES, with values in the range between 60% and 70% for $167.5 \leq \mathrm{M_{top}} \leq 177.5$ and $-2 \leq \Delta\mathrm{JES} \leq +2$. Therefore, as they appear in the likelihood function described in section VII, also their values must be parametrized and this is done by polynomial functions.

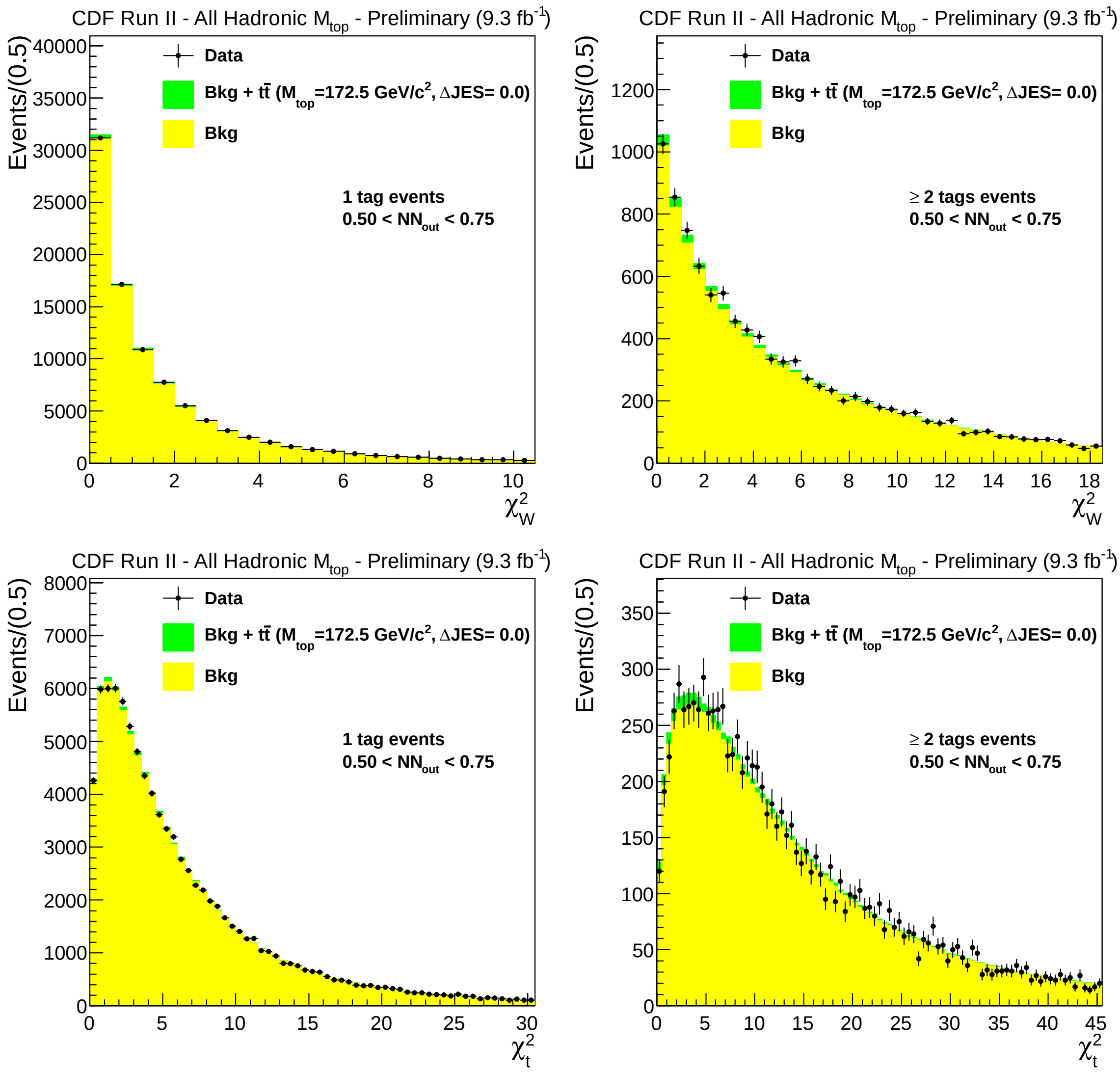

FIG. 2: Distributions of the χ^2 of the fit used to build the m_W^{rec} (upper plots) and m_t^{rec} (lower plots) templates, are shown in a control region defined by $0.50 \le NN_{out} < 0.75$ both for 1-tag events, left plots and $\ge$ 2-tags events, right plots. Along with the data are plotted the corrected expected background and the signal contribution. We see that the agreement is generally good.

VII. LIKELIHOOD

The simultaneous measurement of the top quark mass and the jet energy scale by the template method (TMT2D) consists in finding the values of $\mathrm{M_{top}}$, JES, and the number of signal (n_s) and background (n_b) events for each tagging category which best reproduce the observed distributions of m_t^{rec} and m_W^{rec}, as reconstructed in the selected data samples, given the p.d.f.'s expected for signal and background.

This is done by performing a fit where a likelihood function is maximized, or, equivalently, its negative logarithm is minimized. This function is divided into 3 main parts: the first two terms are the ones strictly needed for the $\mathrm{M_{top}}$ and the JES *in situ* measurements, where the probability for the observed distributions are calculated as a function of eight free parameters ($\mathrm{M_{top}}$, ΔJES, n_s^{1tag}, n_b^{1tag}, $\mathcal{A}_b^{1\,tag}$, $n_s^{\ge 2tags}$, $n_b^{\ge 2tags}$, $\mathcal{A}_b^{\ge 2\,tags}$) for the two tagging categories, while the third one constrains the JES parameter to the *a priori* independent measurement [4] (i.e. $\Delta\mathrm{JES} = 0$ in our notation) to reduce the uncertainty on this variable.

Namely the likelihood, $\mathcal{L}$, is written as:

$$\mathcal{L} = \mathcal{L}_{1\,tag} \times \mathcal{L}_{\ge 2\,tags} \times \mathcal{L}_{\Delta\mathrm{JES}_{constr}}$$

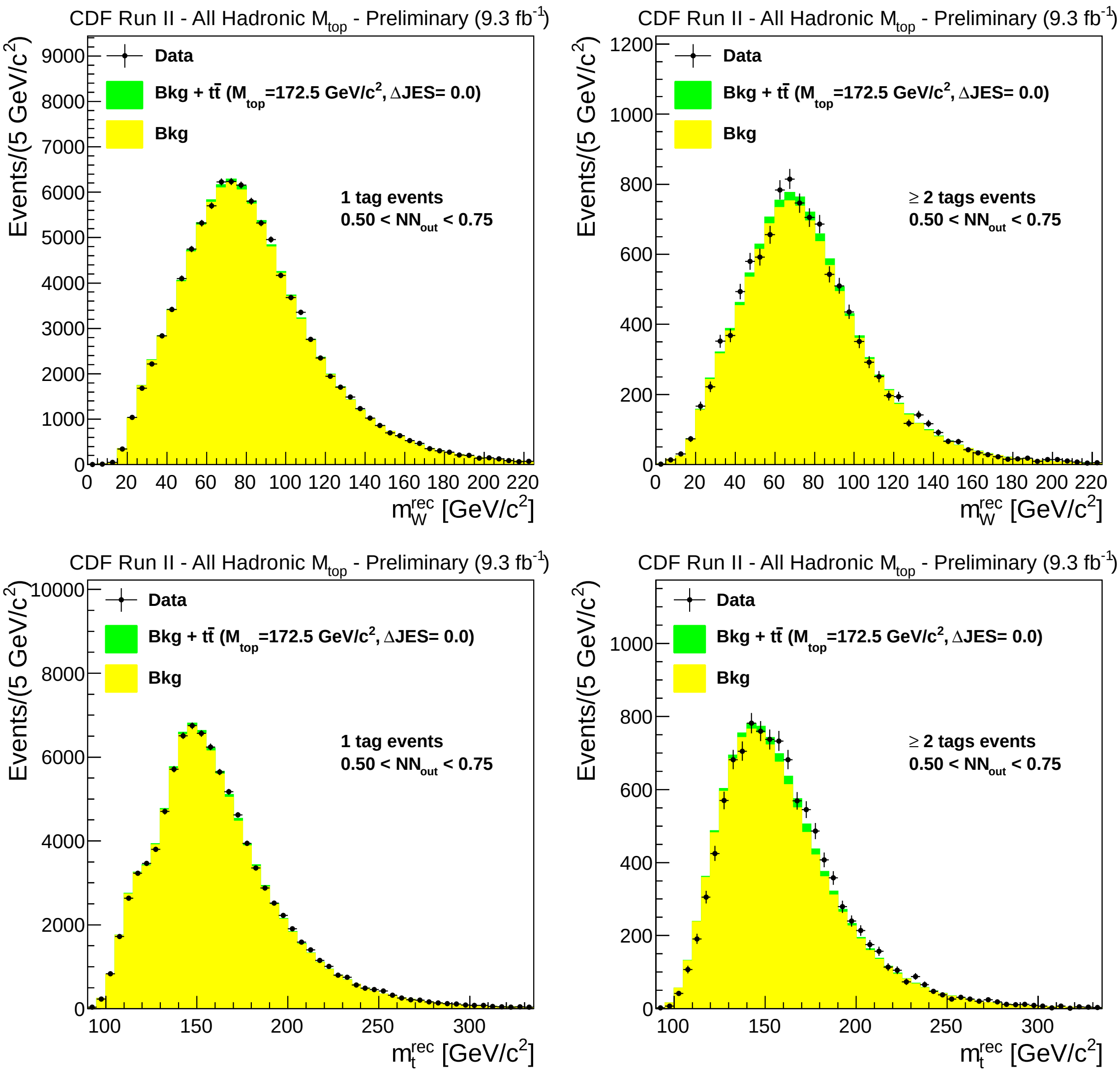

FIG. 3: Templates of the reconstructed W mass, m_W^{rec} (upper plots), and top quark mass, m_t^{rec} (lower plots), are shown in a control region defined by $0.50 \leq NN_{out} < 0.75$ both for 1-tag events, left plots, and $\geq$ 2-tags events, right plots. Along with the data are plotted the corrected expected background and the signal contribution. We see that the agreement is generally good.

The $\mathcal{L}_{1,\geq 2\,tags}$ terms further consist of other factors:

$$\mathcal{L}_{1,\geq 2\,tags} = \mathcal{L}_{\Delta\mathrm{JES}} \times \mathcal{L}_{\mathrm{M_{top}}} \times \mathcal{L}_{evts}$$

where the three terms assume the following form (the superscripts referring to the tag sample are omitted):

CDF Run II - All Hadronic $\mathrm{M_{top}}$ - Preliminary ($9.3\,\mathrm{fb}^{-1}$)

Sample		$N_{\rm obs}$	Expected $t\bar{t}$ (S) ($\mathrm{M_{top}} = 172.5\,\mathrm{GeV}/c^2,\ \Delta\mathrm{JES} = 0$)	Expected Background (B) ($N_{\rm obs} - S$)	S/B
1-tag	S_{JES}	7890	1886 ± 150	6004 ± 174	1 / 3.2
	S_{Mtop}	4130	1270 ± 101	2860 ± 120	1 / 2.2
$\geq$ 2-tags	S_{JES}	1758	782 ± 64	976 ± 77	1 / 1.2
	S_{Mtop}	901	514 ± 42	387 ± 52	1 / 0.75

TABLE I: Numbers of observed data ($N_{\rm obs}$) and expected amount of signal and background events in the samples selected for this analysis. The signal-to-background ratios (S/B) are also shown.

$$
\begin{aligned}
\mathcal{L}_{\Delta\mathrm{JES}} &= \prod_{i=1}^{N_{\rm obs}^{S_{\rm JES}}} \frac{n_s \cdot P_s^{m_W^{rec}}(m_{W,\,i}\,|\,\mathrm{M_{top}},\,\Delta\mathrm{JES}) + n_b \cdot P_b^{m_W^{rec}}(m_{W,\,i})}{n_s + n_b} \\
\mathcal{L}_{\mathrm{M_{top}}} &= \prod_{i=1}^{N_{\rm obs}^{S_{\rm M_{top}}}} \frac{\mathcal{A}_s(\mathrm{M_{top}},\,\Delta\mathrm{JES}) \cdot n_s \cdot P_s^{m_t^{rec}}(m_{t,\,i}\,|\,\mathrm{M_{top}},\,\Delta\mathrm{JES}) + \mathcal{A}_b \cdot n_b \cdot P_b^{m_t^{rec}}(m_{t,\,i})}{\mathcal{A}_s(\mathrm{M_{top}},\,\Delta\mathrm{JES}) \cdot n_s + \mathcal{A}_b \cdot n_b} \\
\mathcal{L}_{evts} &= \sum_{r_s + r_b = N_{obs}^{S_{JES}}} P(r_s,\,n_s) \cdot P(r_b,\,n_b) \cdot \left[\sum_{\substack{t_s \leq r_s,\ t_b \leq r_b \\ t_s + t_b = N_{\rm obs}^{S_{\rm M_{top}}}}} B\,(t_s,\,r_s,\,\mathcal{A}_s) \cdot B\,(t_b,\,r_b,\,\mathcal{A}_b) \right]
\end{aligned}
\tag{1}
$$

In the first term the probability to observe the set $m_{W,\,i},\ (i = 1, ..., N_{\rm obs}^{S_{\rm JES}})$ of m_W^{rec} values reconstructed in the data JES-sample is calculated by the signal and background p.d.f.'s, $P_s^{m_W^{rec}}$ and $P_b^{m_W^{rec}}$ respectively, as a function of the free parameters of the fit. In the second the same is done for the distributions of the observed reconstructed top masses in the $\mathrm{M_{top}}$-sample $m_{t,\,i},\ (i = 1, ..., N_{\rm obs}^{S_{\rm M_{top}}})$, and the m_t^{rec} p.d.f.'s. The third term, $\mathcal{L}_{evts}$, gives the probability to observe simultaneously the number of events selected in the data for the JES-sample and the $\mathrm{M_{top}}$-sample, given the assumed values for the average number of signal (n_s) and background (n_b) events to be expected in $S_{\rm JES}$ and the acceptances $\mathcal{A}_s(\mathrm{M_{top}},\,\Delta\mathrm{JES})$ and $\mathcal{A}_b$. It depends on the Poisson (P) and Binomial (B) probabilities

$$
\begin{aligned}
P(r,\,n) &= \frac{e^{-n} \cdot n^r}{r\,!} \\
B\,(t,\,r,\,\mathcal{A}) &= \binom{r}{t} \cdot \mathcal{A}^t \cdot (1 - \mathcal{A})^{r-t}
\end{aligned}
$$

Finally, $\mathcal{L}_{\Delta\mathrm{JES}_{constr}}$ is a Gaussian term constraining the parameter JES to the value measured and reported in [4], which is equivalent, in our notation, to constrain the parameter ΔJES to 0 within a ± 1 ucertainty :

$$
\begin{aligned}
\mathcal{L}_{\Delta\mathrm{JES}_{constr}} &= e^{-\frac{[\Delta\mathrm{JES}]^2}{2}} \\
&= e^{-\frac{[\Delta\mathrm{JES} - \Delta\mathrm{JES}_{constr}]^2}{2}}
\end{aligned}
$$

In order to facilitate the computation, we minimize the negative logarithm of the likelihood using MINUIT. The uncertainties on the parameters are given by MINOS taking positive and negative statistical error as the difference

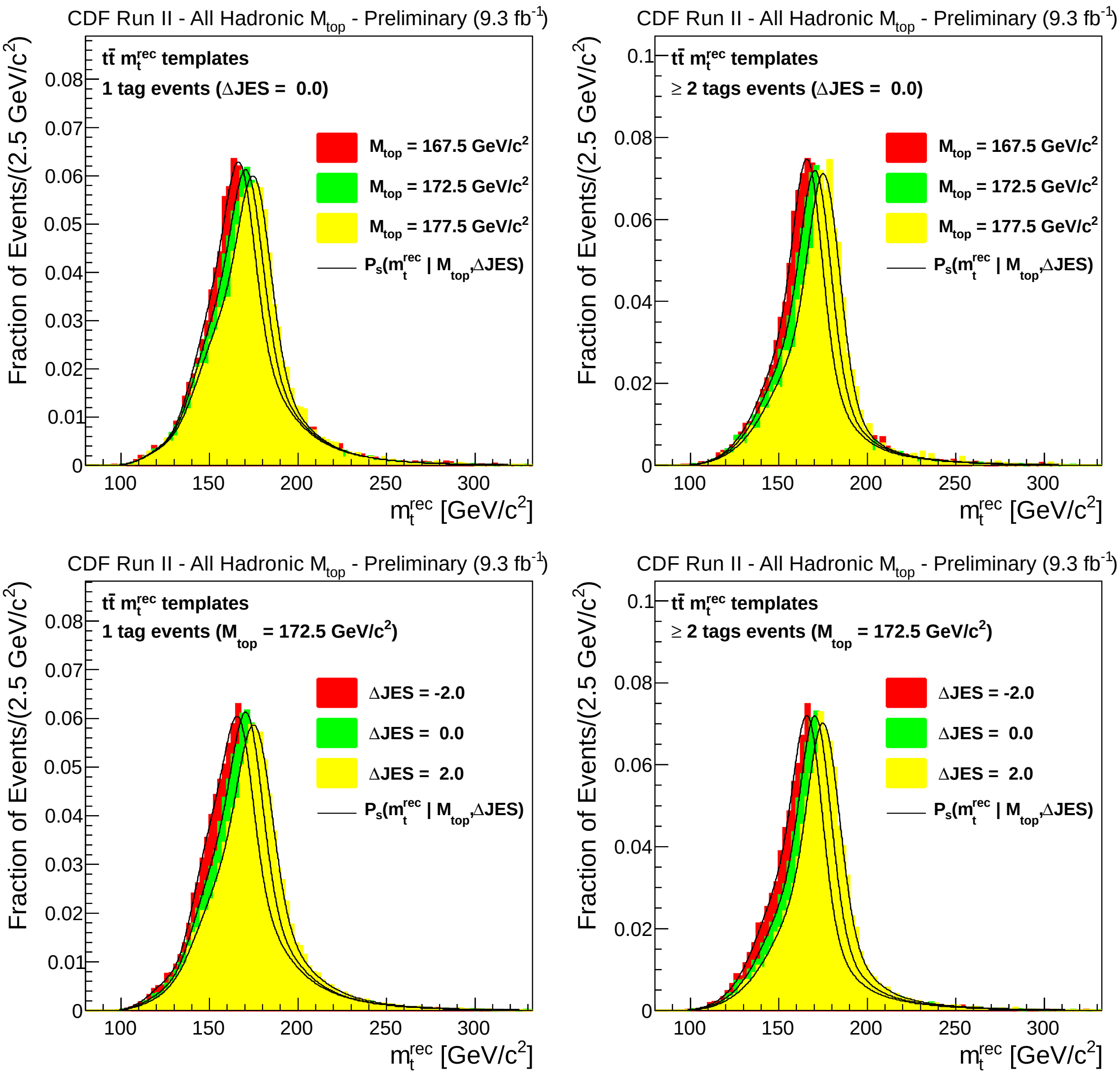

FIG. 4: Probability density functions for the signal m_t^{rec} templates for 1-tag (left plots), and $\geq$ 2-tags events (right plots) for a constant ΔJES value (ΔJES $= 0$), but varying the input top quark mass (upper plots) and for a constant $\mathrm{M_{top}}$ value ($172.5\,\mathrm{GeV}/c^2$), but varying the input jet energy scale (lower plots).

between the observable (O) central value and the values O^+ and O^- for which stands the relation $-lnL(O^{\pm}) + lnL(O) = -1/2$. Following [8] we then take as unique, symmetric errors the average between O^+ and O^- for each parameter. By construction, the MINOS uncertainties take into account the correlations among all the parameters, so that the error on each fitted variable includes both the statistical contribution and the systematic one due to the uncertainties on the other parameters.

VIII. SANITY CHECKS AND EXPECTED PERFORMANCE

We want to investigate the possible presence of biases in the top mass and jet energy scale measurements introduced by our method, as well as to have an estimate of the TMT2D method statistical power before performing the measurement on the actual data sample. To do so, we run realistic pseudo-experiments where "pseudo-data" are extracted from simulated signal and data-driven background templates corresponding to known values of $\mathrm{M_{top}}$ and ΔJES ($\mathrm{M_{top}^{\mathit{in}}}$, $\Delta\mathrm{JES}^{in}$) and used as inputs to the likelihood fit to perform the measurement. Also the other parameters of the fit, i.e. the average numbers of input signal and background events and the background acceptances $\mathcal{A}_b$, are

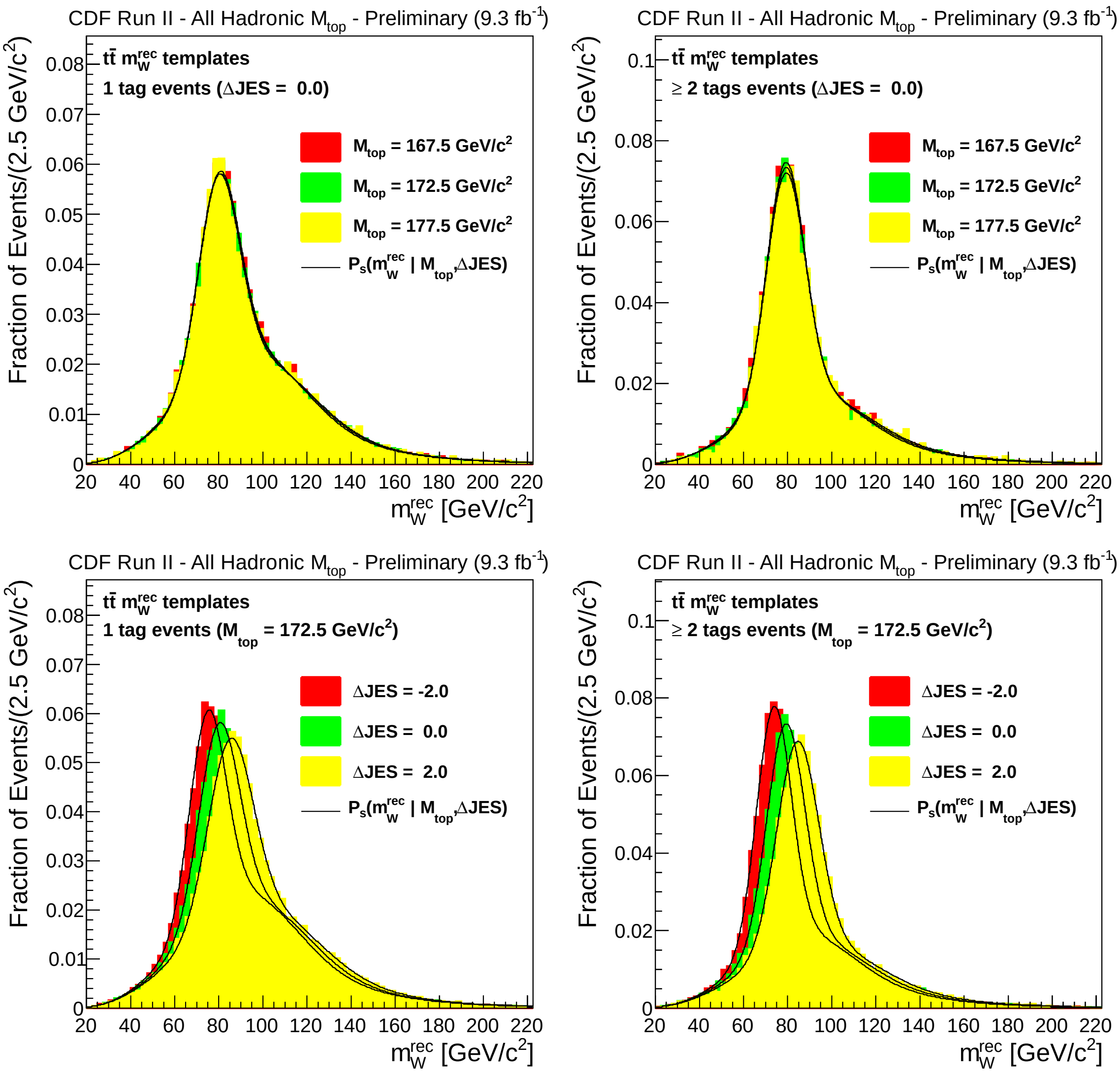

FIG. 5: Probability density functions for the signal m_W^{rec} templates for 1-tag (left plots), and $\geq$ 2-tags events (right plots) for a constant ΔJES value (ΔJES $= 0$), but varying the input top quark mass (upper plots) and for a constant $\mathrm{M_{top}}$ value (172.5 GeV$/c^2$), but varying the input jet energy scale (lower plots).

modified. The results obtained from the fit can then be compared to the true values of the input parameters to study the behavior of the machinery.

A. Pseudo-experiments setup

Sets of about 1000 PEs have been performed at many "points" in the two-dimensional space of the $\mathrm{M_{top}}$ and ΔJES fit parameters. For each point the n_s input values (n_s^{in}) are evaluated as a function of the $\{\mathrm{M_{top}^{\mathit{in}}},\, \Delta\mathrm{JES}^{in}\}$ point by the theoretical $t\bar{t}$ cross section [6], the integrated luminosity of the data sample and the efficiencies of the event selection at that point. The input values n_b^{in} are then taken as the difference between the number of events observed in the data JES-sample and n_s^{in}.
This way, the average total number of events in a PE is kept constant to the observed number of events, while the fractions of signal and background change as a function of the input parameters.
The procedure is the same for each PE:

1. For each tagging category we generate the actual number $N_{(s,\,\mathrm{obs})}^{S_{\mathrm{JES}}}$ of signal events in the JES-sample by a Poisson

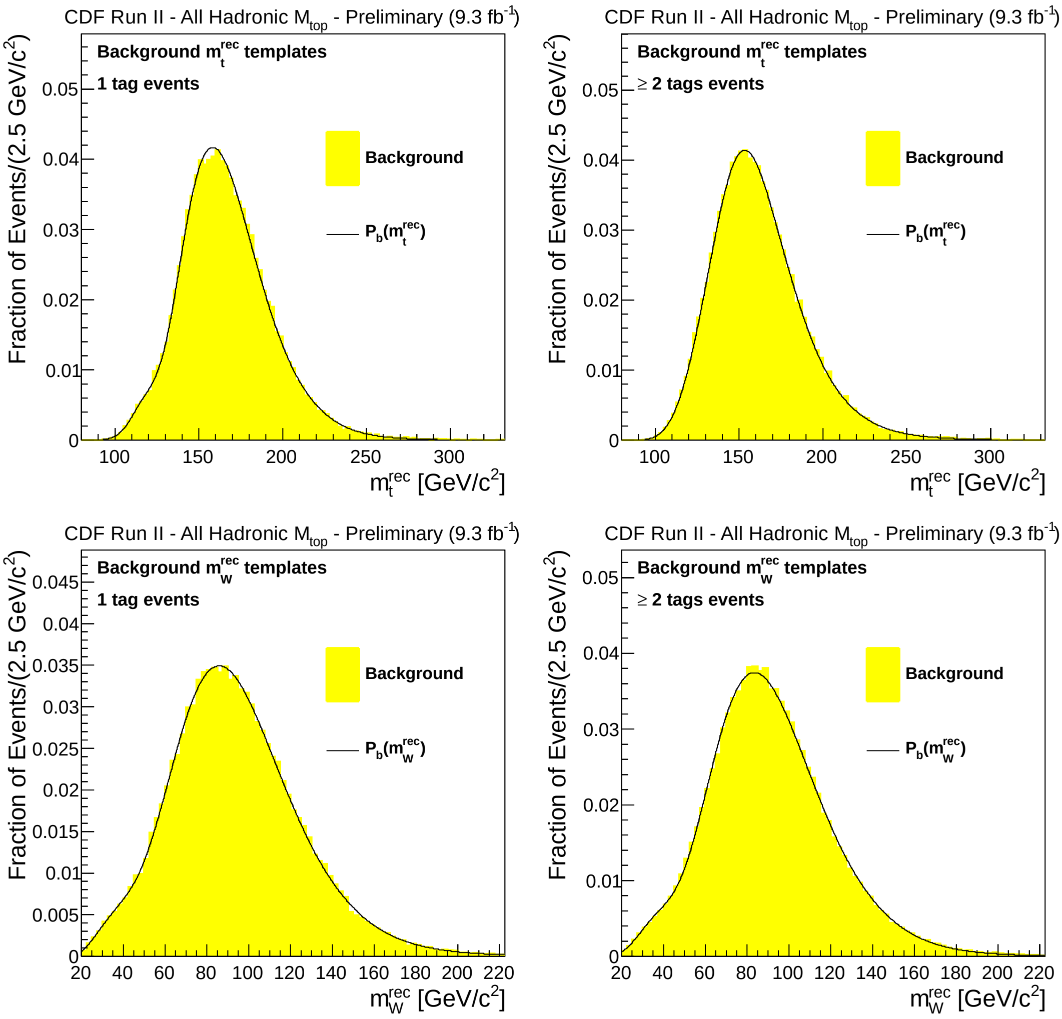

FIG. 6: Background m_t^{rec} (upper plots) and m_W^{rec} (lower plots) templates with the corresponding fitted p.d.f. for 1-tag events, left plots, and $\geq$ 2-tags events, right plots.

distribution with mean n_s^{in}, i.e. the n_s input value; the same is repeated for the actual number of background events $N_{(b,\,\mathrm{obs})}^{S_{\mathrm{JES}}}$ by using a Poisson with mean n_b^{in}.

2. The input values of the signal acceptances $\mathcal{A}_s^{in}$ are evaluated directly from the Monte Carlo samples corresponding to the input $\{M_{\mathrm{top}}^{in}, \Delta\mathrm{JES}^{in}\}$, while, as it concerns the input $\mathcal{A}_b^{in}$, they are set evaluating the numbers of expected background events both in the JES and the $\mathrm{M_{top}}$-samples, using always the difference between the observed data and the expected signal, as explained for n_b^{in} above.

3. The number of signal events in the $\mathrm{M_{top}}$-sample, $N_{(s,\,\mathrm{obs})}^{S_{\mathrm{M_{top}}}}$, is generated by a Binomial distribution, given $N_{(s,\,\mathrm{obs})}^{S_{\mathrm{JES}}}$ and the acceptance $\mathcal{A}_s$. Again, the same procedure is repeated for the background, to obtain $N_{(b,\,\mathrm{obs})}^{S_{\mathrm{M_{top}}}}$, obviously using $N_{(b,\,\mathrm{obs})}^{S_{\mathrm{JES}}}$ and $\mathcal{A}_b$.

4. For each signal event, corresponding reconstructed masses must be considered with average distributions given by the signal templates. In particular we have one m_W^{rec} value for each event in S_{JES} and one m_t^{rec} value for each event in $S_{\mathrm{M_{top}}}$. More precisely, as being $S_{\mathrm{M_{top}}} \subseteq S_{\mathrm{JES}}$, a pair of values for m_W^{rec} and m_t^{rec} exist for each event in the $\mathrm{M_{top}}$-sample, while for events belonging to S_{JES} *but NOT* to $S_{\mathrm{M_{top}}}$ one has a single value for m_W^{rec} only.

   Then, to take into account correlations between m_t^{rec} and m_W^{rec} in the same event, $N_{(s,\,\mathrm{obs})}^{S_{\mathrm{M_{top}}}}$ m_W^{rec} and m_t^{rec} values are both drawn from signal two-dimensional histograms where m_W^{rec} *vs* m_t^{rec} are plotted for each event in $S_{\mathrm{M_{top}}}$. Finally, the missing $N_{(s,\,\mathrm{obs})}^{S_{\mathrm{JES}}} - N_{(s,\,\mathrm{obs})}^{S_{\mathrm{M_{top}}}}$ values of m_W^{rec} are drawn from distributions of m_W^{rec} obtained from events belonging to S_{JES} *but NOT* to $S_{\mathrm{M_{top}}}$ (this set is simply denoted by $S_{\mathrm{JES}} - S_{\mathrm{M_{top}}}$ in the following). Obviously all the histograms used here correspond to the input values $\{\mathrm{M}_{\mathrm{top}}^{in}, \Delta\mathrm{JES}^{in}\}$.

5. The same procedure just outlined for the signal events is repeated to generate the $N_{(b,\,\mathrm{obs})}^{S_{\mathrm{JES}}}$ and $N_{(b,\,\mathrm{obs})}^{S_{\mathrm{M_{top}}}}$ m_W^{rec} and m_t^{rec} values respectively by the background templates. We remind that these templates are obtained correcting the raw background prediction using numbers and signal shapes corresponding to the input values of the fit parameters.

6. The actual value of $\Delta\mathrm{JES}_{constr}$ to be used in the term $\mathcal{L}_{\Delta\mathrm{JES}_{constr}}$ is extracted from a Gaussian of mean $\Delta\mathrm{JES}^{in}$ and width 1;

7. $-\log\mathcal{L}$ is simultaneously minimized with respect to the 8 free parameters, $\mathrm{M_{top}}$, ΔJES, $n_s^{1\,tag}$, $n_b^{1\,tag}$, $\mathcal{A}_b^{1\,tag}$, $n_s^{\geq 2\,tags}$, $n_b^{\geq 2\,tags}$ and $\mathcal{A}_b^{\geq 2\,tags}$.

Histograms are filled by outputs from each PE and then used to study the average behavior of the measurement machinery with respect to the true input quantities. Uncertainties on variables extracted from these histograms and related to the limited statistic of the samples used to build the templates[14], are evaluated by a *bootstrap* procedure [9, 10], that is fluctuating the contents of each bin in the templates by its statistical uncertainty and performing PEs extracting data from the set of "fluctuated" templates. This is repeated 100 times, and the RMS of variables extracted from histograms are taken as their statistical uncertainties.

B. Calibration

There are many factors which may introduce a bias using the TMT2D method like e.g. unappropriate parametrizations of the templates by smooth p.d.f.'s. We take advantage of the PEs procedure to find calibration functions to be applied to the outputs of a measurement to obtain, on the average, more reliable estimates of the true input values of the fitted parameters. As it concerns in particular $\mathrm{M_{top}}$ and ΔJES, the calibrated values will be denoted by $\mathrm{M}_{\mathrm{top}}^{corr}$ and $\Delta\mathrm{JES}^{corr}$ respectively. Obviously, also the uncertainties from the likelihood fit have to be propagated through the calibration. To test the goodness of the calibration we performed a complete set of PEs where it is applied PE by PE. In Fig. 7 we show examples of the residuals of $\mathrm{M_{top}}$ and ΔJES after the calibration. These plots show how the applied corrections get rid of most of the average biases.

To check that the calibrated uncertainties, $\sigma_{\mathrm{M_{top}}}^{corr}$ and $\sigma_{\Delta\mathrm{JES}}^{corr}$, are unbiased we consider the width of $\mathrm{M}_{\mathrm{top}}^{corr}$ and $\Delta\mathrm{JES}^{corr}$ pull distributions [15]. Fig. 8 shows examples of the values of the $\mathrm{M_{top}}$ and ΔJES pull widths as a function of the input top mass, $\mathrm{M}_{\mathrm{top}}^{in}$ and of the input ΔJES, $\Delta\mathrm{JES}^{in}$, after the calibration. To derive a correction we average the pull widths over all the $\mathrm{M}_{\mathrm{top}}^{in}$ and $\Delta\mathrm{JES}^{in}$ values. This procedure leads to multiplicative correction factors equal to 1.024 for $\sigma_{\mathrm{M_{top}}}^{corr}$ and to 1.004 for $\sigma_{\Delta\mathrm{JES}}^{corr}$.

Figures 9 shows examples of the expected uncertainties after both the calibration and the pull width correction have been applied. The values of these average expected uncertainty on the measured top quark mass and jet energy scale displacement for true Mtop and ΔJES around $172.5\,\mathrm{GeV}/c^2$ and 0, are:

$$\begin{aligned} \sigma_{\mathrm{M_{top}}}^{corr}\,(stat + \mathrm{Fit\ syst}) &\simeq 1.35\,\mathrm{GeV}/c^2 \\ \sigma_{\Delta\mathrm{JES}}^{corr}\,(stat + \mathrm{Fit\ syst}) &\simeq 0.30 \end{aligned}$$

These uncertainties actually include the systematic contributions due to all the parameters of the fit.

IX. SYSTEMATIC UNCERTAINTIES ON THE TOP QUARK MASS AND THE JET ENERGY SCALE

Various sources of systematic uncertainties might affect the top quark mass and the jet energy scale measurements. The main possible effect have been studied and are summarized in this section.

They are usually evaluated by performing PEs extracting pseudo-data from templates built using signal samples where the possible systematic effects have been considered and included. Corresponding corrections to the shape of raw background templates are applied to obtain also the corrected background templates in agreement with the effect

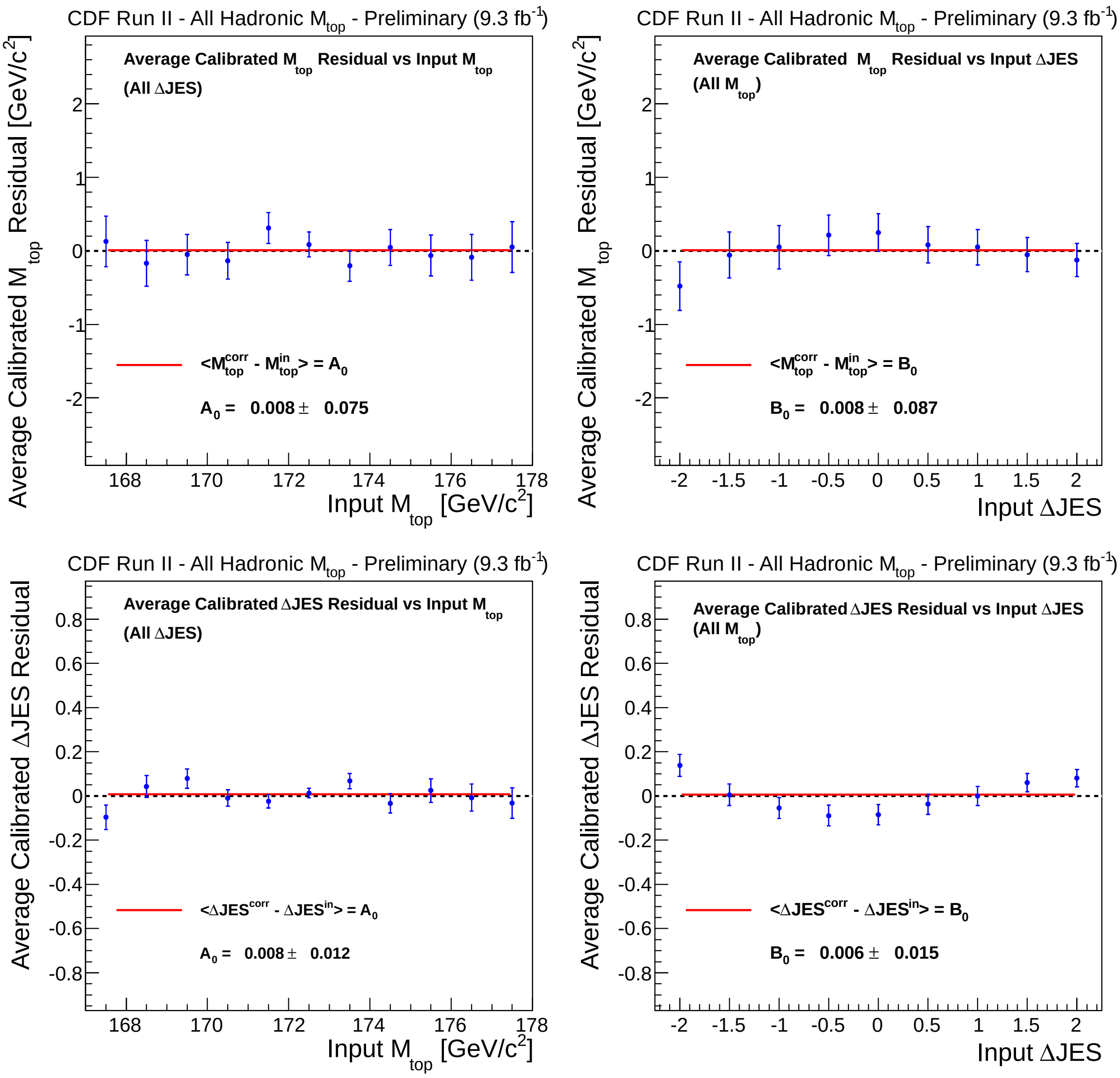

FIG. 7: Residuals of the calibrated top quark mass ($M_{top}^{corr} - M_{top}^{in}$, upper plots) and jet energy scale displacement ($\Delta JES^{corr} - \Delta JES^{in}$, lower plots) as a function of the input M_{top} (left plots), and of input ΔJES (right plots). The results of fits by a straight line are superimposed. In the left (right) plots each point at a given M_{top}^{in} (ΔJES^{in}) is in turn an average over all the ΔJES^{in} (M_{top}^{in}) values.

one wants to study. On the contrary, nothing is changed in the measurement machinery, i.e. in the elements of the likelihood fit, because it is this machinery that we want to apply to the data and that, therefore, we have to test in front of possible mismodeling of the data themeselves.

The results from these PEs are then compared to the ones obtained by using default templates, and the shifts in the average M_{top}^{corr} and ΔJES^{corr} values are used to estimate of the systematic uncertainty. In some cases the statistical uncertainty on the shifts may be larger than the shifts themselves and therefore we use conservatively the former as systematic uncertainty.

a. **Residual Bias / Calibration** The calibration gets rid of the *average* biases, related especially to the templates parametrization by smooth probability density functions. However, residual biases usually exist at individual $\{M_{top}^{in}, \Delta JES^{in}\}$ points, and have to be taken into account. Similarly to what done to define a correction for the calibrated uncertainties in section VIII B, to evaluate the residual bias we consider the *mean* of pull distributions at all different $\{M_{top}^{in}, \Delta JES^{in}\}$ points. Examples of pull means are shown in figure 10.

To consider properly the local biases, we perform separate averages of positive and negative pull means. This leads

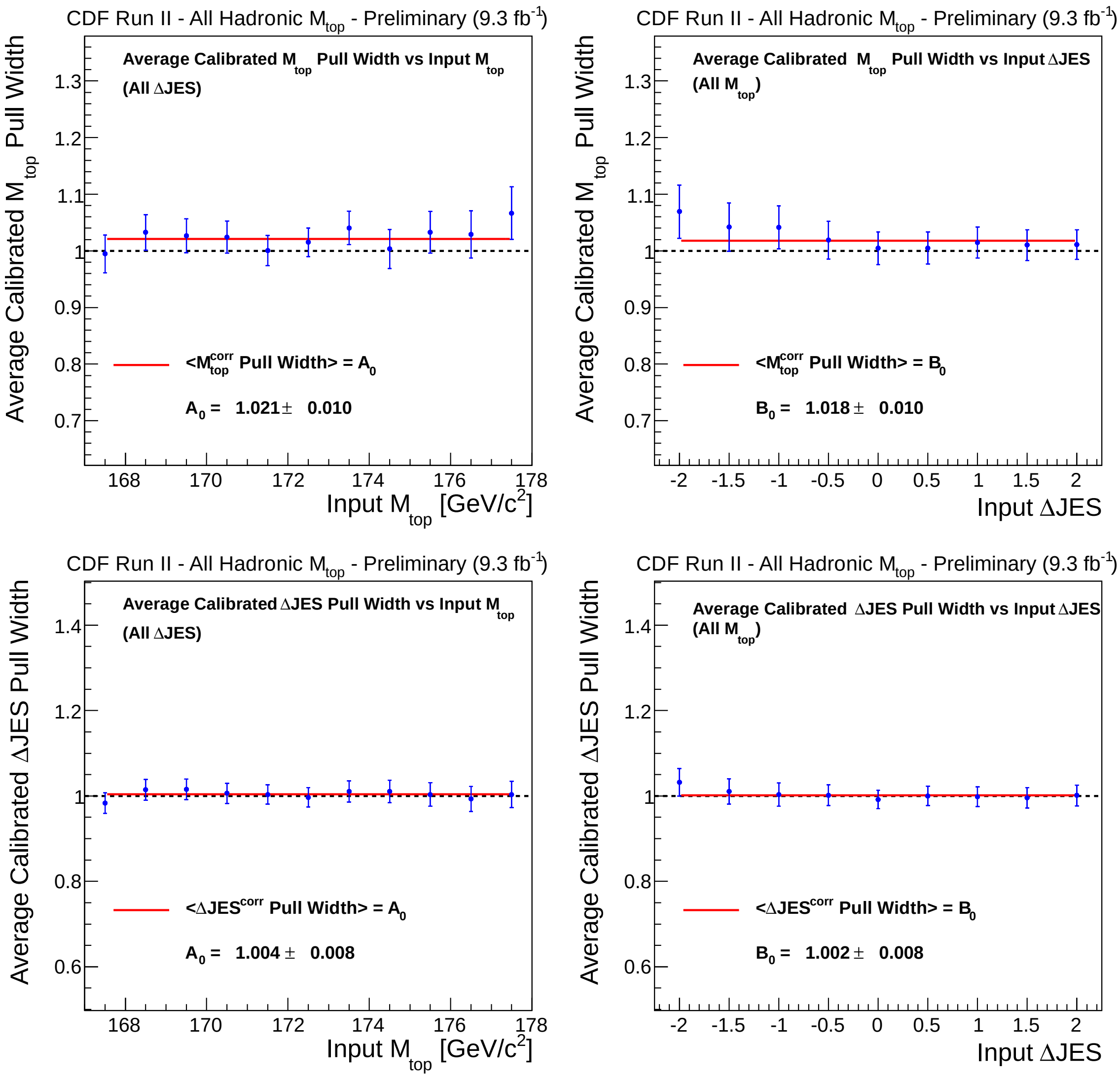

FIG. 8: Widths of the pull distributions for $M_{\rm top}^{corr}$ (upper plots) and $\Delta{\rm JES}^{corr}$ (lower plots) as a function of the input $M_{\rm top}$ (left plots) and of the input ΔJES (right plots). The straight lines denote the fit by a constant function. In the left (right) plots each point at a given $M_{\rm top}^{in}$ ($\Delta{\rm JES}^{in}$) is in turn an average over all the $\Delta{\rm JES}^{in}$ ($M_{\rm top}^{in}$) values

to

$$\sigma^{syst}_{M_{\rm top}}\,(\text{Res Bias / Calibration}) \;\simeq\; \left({}^{+0.172}_{-0.153}\right)\cdot\sigma^{corr}_{M_{\rm top}}\,(stat + Fit\ syst)$$
$$\sigma^{syst}_{\Delta{\rm JES}}\,(\text{Res Bias / Calibration}) \;\simeq\; \left({}^{+0.233}_{-0.291}\right)\cdot\sigma^{corr}_{\Delta{\rm JES}}\,(stat + Fit\ syst)$$

This means that, at central points like $\left\{M_{\rm top}^{in} = 172.5\ {\rm GeV}/c^2,\ \Delta{\rm JES}^{in} = 0\right\}$, systematic "residual bias and calibration" uncertainties of about $\left({}^{+0.23}_{-0.21}\right)$ GeV$/c^2$ for $M_{\rm top}$ and $\left({}^{+0.071}_{-0.088}\right)$ for ΔJES may be expected.

b. **Generator (hadronization)** Many sources of systematic effects arise from uncertainties in the Monte Carlo modeling of the hard interaction and the hadronization process, where the latter contribution has been verified to be the more important. To estimate this uncertainty we use samples generated using PYTHIA and HERWIG Monte Carlo generators, which differ in their hadronization schemes and in their description of the underlying event and multiple interactions.

Templates are built using events from these samples (at $M_{\rm top}^{in} = 172.5\,{\rm GeV}/c^2$, $\Delta{\rm JES}^{in} = 0$) and PEs are performed drawing pseudo-data from these distributions. By considering the shift between the two samples, the estimated systematic uncertainties due to this source are $\sigma^{syst}_{M_{\rm top}}$ (Generator) $\simeq 0.29\,{\rm GeV}/c^2$ and $\sigma^{syst}_{\Delta{\rm JES}}$ (Generator) $\simeq 0.273$.

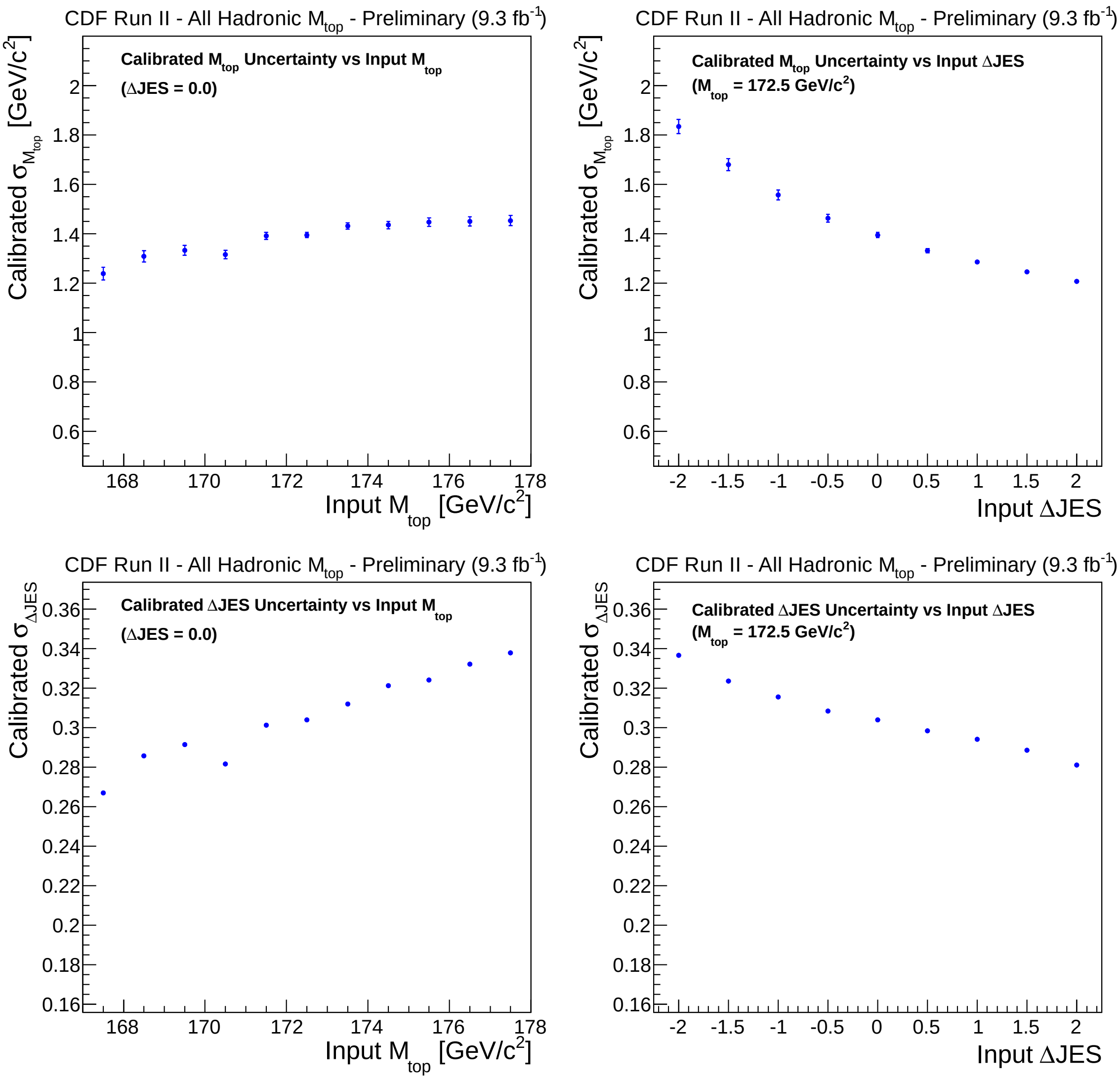

FIG. 9: The expected uncertainties on the top mass ($\sigma_{\mathrm{M_{top}}}$ $(stat$ + Fit Syst)) and on the jet energy scale displacement ($\sigma_{\Delta\mathrm{JES}}$ $(stat$ + Fit Syst)) are shown as a function of $\mathrm{M}^{in}_{\mathrm{top}}$ (at constant ΔJES = 0, left) and of $\Delta\mathrm{JES}^{in}$ for $\mathrm{M}^{in}_{\mathrm{top}} = 172.5\,\mathrm{GeV}/c^2$ (right), after both the calibration and the pull width corrections have been applied.

c. **Initial and Final State Radiation (ISR / FSR)** Additional jets coming from possible emission of hard gluons might fall among the six leading jets and populate the tails in the top quark invariant mass distribution. The amount of radiation from partons in the initial (ISR) or final (FSR) state is set by parameters of the generators used to simulate signal events. To study effects of uncertainties on those parameters, templates are built using samples where their values have been changed with respect to the default, to increase or to decrease the amount of radiation. Again, PEs are performed where pseudo-data are drawn from these modified templates and the results compared to the default. The resulting uncertainties are $\sigma^{syst}_{\mathrm{M_{top}}}$ (ISR/FSR) $\simeq 0.13\,\mathrm{GeV}/c^2$ and $\sigma^{syst}_{\Delta\mathrm{JES}}$ (ISR/FSR) $\simeq 0.232$.

d. ***b*-jets Energy Scale** Since the default jet energy corrections are derived using data samples deprived of heavy flavors, an additional uncertainty comes from considering the different properties of b quarks. We account for the uncertainties due to the b-quark semileptonic branching ratios, the fragmentation modeling, and the response of the calorimeters to b and c hadrons. Templates are built varying the default assumption for the three mentioned sources, and PEs are performed drawing pseudo-data from these modified distributions. The comparison to the default results gives systematic uncertainties $\sigma^{syst}_{\mathrm{M_{top}}}$ ($b-$JES) $\simeq 0.20\,\mathrm{GeV}/c^2$ and $\sigma^{syst}_{\Delta\mathrm{JES}}$ (($b-$JES) $\simeq 0.035$.

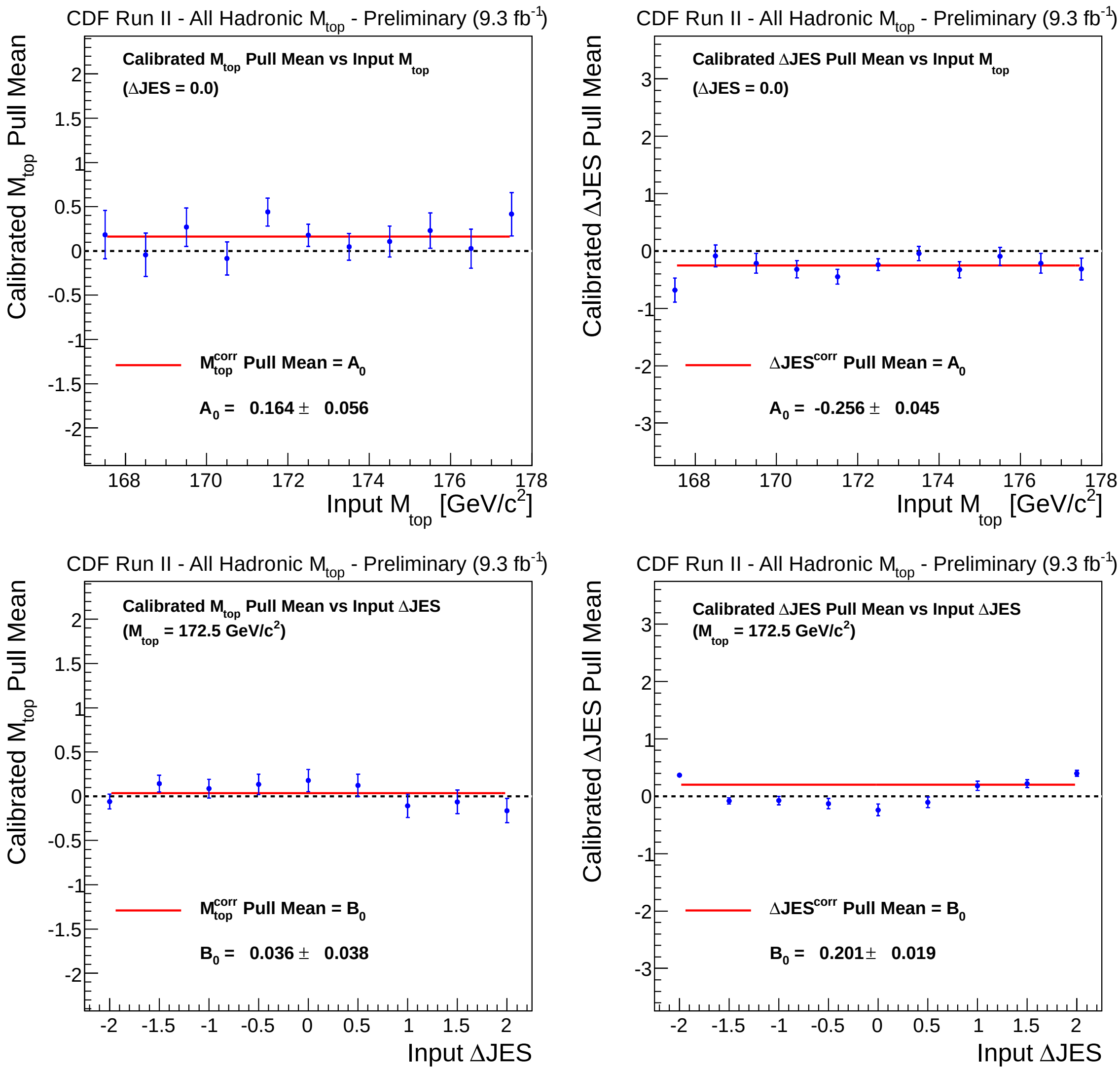

FIG. 10: $\mathrm{M_{top}}$ (on the left) and ΔJES (on the right) pull means as a function of $\mathrm{M}^{in}_{\mathrm{top}}$ at constant $\Delta\mathrm{JES} = 0$ (upper plots) and as a function of $\Delta\mathrm{JES}^{in}$ for $\mathrm{M}^{in}_{\mathrm{top}} = 172.5\,\mathrm{GeV}/c^2$ (lower plots). The straight lines denote the fit by a constant function.

e. ***b*-tagging** The different efficiency of the b-tagging algorithm on data and Monte Carlo simulated events is usually considered a constant Scale Factor (b-tag SF). However this value might have a dependence on the transverse energy of jets, leading to possible variations in the shapes of m_t^{rec} and m_W^{rec} templates. Since the background estimate is data-driven, the analysis is sensitive to an overall uncertainty in the b-tagging scale factor only through signal shapes. Signal templates are built taking into account the possible dependence of the SF on the jet E_T and then used in PEs. The corresponding systematic effects have been evaluated to be $\sigma_{\mathrm{M}^{syst}_{\mathrm{top}}}\,(b\mathrm{-tagging}) \simeq 0.04\,\mathrm{GeV}/c^2$ and $\sigma^{syst}_{\Delta\mathrm{JES}}\,(b\mathrm{-tagging}) \simeq 0.018$.

f. **Residual JES** Our templates are built displacing the value of the jet energy scale by fractions of its uncertainty σ_{JES}, as estimated in [4]. However σ_{JES} results from many independent effects with different behavior with respect to properties of jets like E_T and η, and represents therefore a leading order estimate. So, second order effects can arise from uncertainties on single levels of correction of the jet energies. To evaluate these possible effects, we build signal templates by varying separately by $\pm 1\,\sigma$ the single corrections and PEs were then performed by using these templates and not applying the constraint $\mathcal{L}_{\Delta\mathrm{JES}_{constr}}$ in the likelihood fit. The resulting uncertainties have been added in quadrature to obtain a "Residual JES" uncertainty on the top mass: $\sigma^{syst}_{\mathrm{M_{top}}}\,(\mathrm{Residual\,JES}) \simeq 0.57\,\mathrm{GeV}/c^2$

g. **Parton Distribution Functions (PDF)** The choice of parton distribution functions (PDF) inside the proton can affect the kinematics of $t\bar{t}$ events and thus the top quark mass measurement. We estimate the uncertainty resulting from the possible PDF models by reweighting Monte Carlo events by their probability to occur according many different PDF's. Templates are built by weighted events, PEs are performed by extracting pseudo-data from these modified distributions and the shifts in the average $\mathrm{M}_{\mathrm{top}}^{corr}$ and $\Delta\mathrm{JES}^{corr}$ values are taken as systematic uncertainties. The resulting total uncertainties due to parton distributions are $\sigma^{syst}_{\mathrm{M_{top}}}\,(\mathrm{PDF}) \simeq \binom{+0.18}{-0.36}$ GeV/c^2 and $\sigma^{syst}_{\Delta\mathrm{JES}}\,(\mathrm{PDF}) \simeq \binom{+0.096}{-0.052}$.

h. **Pileup** In this analysis, the effects of possible discrepancies between real data and Monte Carlo due to pileup of events and related to changes in the istantaneous luminosity, have been taken into account by reweighting the events of the default Monte Carlo samples in order to reproduce the distribution of the number of primary vertices observed in the data. However possible mismodeling of minimum bias events can still affect the measurement even if the same luminosity profile is used in data and Monte Carlo. The systematic uncertainty from this effect is related to the Residual JES systematic and affects therefore only the top quark mass. It is estimated to amount to $\sigma^{syst}_{\mathrm{M_{top}}}\,(\mathrm{Pileup}) \simeq 0.23\,\mathrm{GeV}/c^2$.

i. **Color Reconnections** Uncertainties from modeling of color reconnections effects [12] are estimated by comparing the results of two sets of PEs performed drawing pseudo-data from templates built by Monte Carlo samples where different tunes of parameters have been set, corresponding to different models of color reconnections. This gives $\sigma^{syst}_{\mathrm{M_{top}}}\,(\mathrm{Color\,Reconn.}) \simeq 0.32\,\mathrm{GeV}/c^2$ and $\sigma^{syst}_{\Delta\mathrm{JES}}\,(\mathrm{Color\,Reconn.}) \simeq 0.101$ for this source of uncertainty.

j. **Templates Statistics** As mentioned in section VIII A, the shapes of signal and background templates are affected by uncertainties due to the limited statistics of the Monte Carlo (for the signal) and data (for the background) samples used to build them. These uncertainties affect the results of a measurement, which is performed by an unbinned likelihood where parametrized p.d.f.' s, fitted to default templates, are evaluated. We address this effect obtaining 100 sets of templates by statistical fluctuations of default ones, and performing pseudo-experiments drawing data from each of these sets separately. The spread in the average values of $\mathrm{M}_{\mathrm{top}}^{corr}$ and $\Delta\mathrm{JES}^{corr}$ distributions through the 100 sets is taken as systematic uncertainty. This was repeated at many ($\mathrm{M}_{\mathrm{top}}^{in}$, $\Delta\mathrm{JES}^{in}$) points and an average gives $\sigma^{syst}_{\mathrm{M_{top}}}\,(\mathrm{Templ.\,Stat.}) \simeq 0.34\,\mathrm{GeV}/c^2$ and $\sigma^{syst}_{\Delta\mathrm{JES}}\,(\mathrm{Templ.\,Stat.}) \simeq 0.071$.

k. **Trigger Simulation** The multijet trigger, used for the first online selection of $t\bar{t}$ candidate events in the data, is simulated on signal Monte Carlo events. Uncertainties on this simulation, possibly related to mismodeling of the energy deposition in the calorimeters and/or changes of the trigger algorithms and requirements not faithfully reproduced in the default Monte Carlo samples, are taken into account. Templates are built by events where the trigger simulation has been modified and PEs performed drawing pseudo-data from them. Comparison to the default PEs leads to uncertainties $\sigma^{syst}_{\mathrm{M_{top}}}\,(\mathrm{Trigger}) \simeq 0.61\,\mathrm{GeV}/c^2$ and $\sigma^{syst}_{\Delta\mathrm{JES}}\,(\mathrm{Trigger}) \simeq 0.188$.

l. **Data Luminosity and $t\bar{t}$ Cross Section** The pseudoexperiments used for defining the calibration of the measurement were performed assuming values for the input mean numbers of signal events n_s^{in} given by

$$n_s^{in} = \sigma_{t\bar{t}}\left(\mathrm{M}_{\mathrm{top}}^{in}\right) \cdot L \cdot \varepsilon\left(\mathrm{M}_{\mathrm{top}}^{in},\, \Delta\mathrm{JES}^{in}\right)$$

where $\sigma_{t\bar{t}}\left(\mathrm{M}_{\mathrm{top}}^{in}\right)$ is the theoretical cross section corrsponding to input top quark mass as calculated in [6], L is the total integrated luminosity of the data sample and $\varepsilon\left(\mathrm{M}_{\mathrm{top}}^{in},\, \Delta\mathrm{JES}^{in}\right)$ is the efficiency of the event selection. Moreover the input values for the acceptances $\mathcal{A}_s$ of the $\mathrm{M_{top}}$ sample w.r.t. the JES sample is also derived from the Monte Carlo.

Uncertainties of these variables affect therefore the amount of signal and background events [16] in the data sample assumed for the calibration. The statistical contribution to this uncertainty is already taken into account by the "Templates Statistics" systematic. Variations of $\pm 1\,\sigma$ have been considered both for the $t\bar{t}$ cross section (as given in [6]) and the integrated luminosity of the data. Pseudoexperiments have then been performed changing the amount of signal and backgorund events according to these variations and the results compared to default. The systematic uncertainties from the two contributions amount to: $\sigma^{syst}_{\mathrm{M_{top}}}\,(\mathrm{Data\,Luminosity}) \simeq 0.15\,\mathrm{GeV}/c^2$, $\sigma^{syst}_{\Delta\mathrm{JES}}\,(\mathrm{Data\,Luminosity}) \simeq 0.032$ and $\sigma^{syst}_{\mathrm{M_{top}}}\,(\sigma_{t\bar{t}}) \simeq 0.15\,\mathrm{GeV}/c^2$, $\sigma^{syst}_{\Delta\mathrm{JES}}\,(\sigma_{t\bar{t}}) \simeq 0.034$.

m. **Background Shape** The background p.d.f.'s used in the likelihood fit, and therefore in the measurement, is based on parametrizations of templates built by applying weights to data in the pretag samples, as outlined in sections III and IV C. The "raw" distributions obtained this way must then be corrected to take into account the presence of $t\bar{t}$ events in the data. Possible effects related to this correction are taken into account by the calibration procedure or other systematic uncertainties. Residual mismodeling of the background shape more directly related to the raw distributions can remain and are treated as systematic uncertainties which amount to $\sigma^{syst}_{\mathrm{M_{top}}}$ (Bkg Shape) $\simeq$ 0.15 GeV/c^2 and $\sigma^{syst}_{\Delta\mathrm{JES}}$ (Bkg Shape) $\simeq$ 0.014.

A. Total systematic uncertainty

Table IX A shows a summary of all the systematic uncertainties and their quadrature sum, which gives a total, symmetrized systematic uncertainty of ±1.15 GeV/c^2 for the $\mathrm{M_{top}}$ measurement and ±0.44 for the ΔJES, where the "Residual Bias / Calibration" uncertainty, depending on errors from the likelihood fit, is already evaluated at the values given by the measurement on the data, described in section X.

CDF Run II - All Hadronic $\mathrm{M_{top}}$ - Preliminary (9.3 fb^{-1})

Source	$\sigma^{syst}_{\mathrm{M_{top}}}$ (GeV/c^2)	$\sigma^{syst}_{\Delta\mathrm{JES}}$
Residual bias / Calibration	$^{+0.27}_{-0.24}$	$^{+0.077}_{-0.096}$
Generator (hadronization)	0.29	0.273
Initial / Final State Radiation	0.13	0.232
b-jets Energy Scale	0.20	0.035
b-tagging	0.04	0.018
Residual Jet Energy Scale	0.57	$--$
Parton Distribution Functions	$^{+0.18}_{-0.36}$	$^{+0.096}_{-0.052}$
Pileup	0.22	0.000
Color Reconnections	0.32	0.101
Templates Statistics	0.34	0.071
Trigger Simulation	0.61	0.188
Data Luminosity	0.15	0.032
$t\bar{t}$ Cross Section	0.15	0.034
Background Shape	0.15	0.014
Total	$^{+1.13}_{-1.17}$	$^{+0.445}_{-0.440}$

TABLE II: Breakdown of *observed* systematic uncertainties from different sources and their respective amount. The contribution depending on the statistical errors (i.e. the "Residual Bias / Calibration") has been calculated here by the values observed in the measurement on data. The total uncertainty is obtained by the quadrature sum of single contributions.

X. THE TOP QUARK MASS MEASUREMENT

After the kinematic selections with $NN_{out} \geq 0.97$ ($NN_{out} \geq 0.94$), $\chi^2_W \leq 2.0$ ($\chi^2_W \leq 3.0$) and $\chi^2_t \leq 3.0$ ($\chi^2_t \leq 4.0$) for events with 1 tag ($\geq$ 2 tags), we are left with 7890 and 4130 events in the JES-sample and $\mathrm{M_{top}}$-sample with 1 tag respectively, and 1758 and 901 events in the corresponding samples with $\geq$ 2 tags. The expected signal, assuming $\mathrm{M_{top}} = 172.5$ GeV/c^2 and $\Delta\mathrm{JES} = 0$, amounts to 1886 ± 150 (1-tag JES-sample), 1270 ± 101 (1-tag $\mathrm{M_{top}}$-sample), 782 ± 64 ($\geq$ 2-tag JES-sample), and 514 ± 42 ($\geq$ 2-tag $\mathrm{M_{top}}$-sample) events. The likelihood fit described in Sec. VII has been applied to the data samples to derive the best top quark mass and jet energy scale displacement from the default value to be

$$\begin{aligned} \mathrm{M}_{\mathrm{top}}^{fit} &= 174.82 \pm 1.25\,(stat + Fit\,syst)\,\mathrm{GeV}/c^2 \\ \Delta\mathrm{JES}^{fit} &= -0.118 \pm 0.262\,(stat + Fit\,syst) \end{aligned}$$

Figure 11 shows the behavior of the likelihood as a function of the $\mathrm{M_{top}}$ and ΔJES parameters and the contours corresponding to variations of one, two and three standard deviations of the same parameters with respect to the values maximizing the likelihood itself (before the calibration).

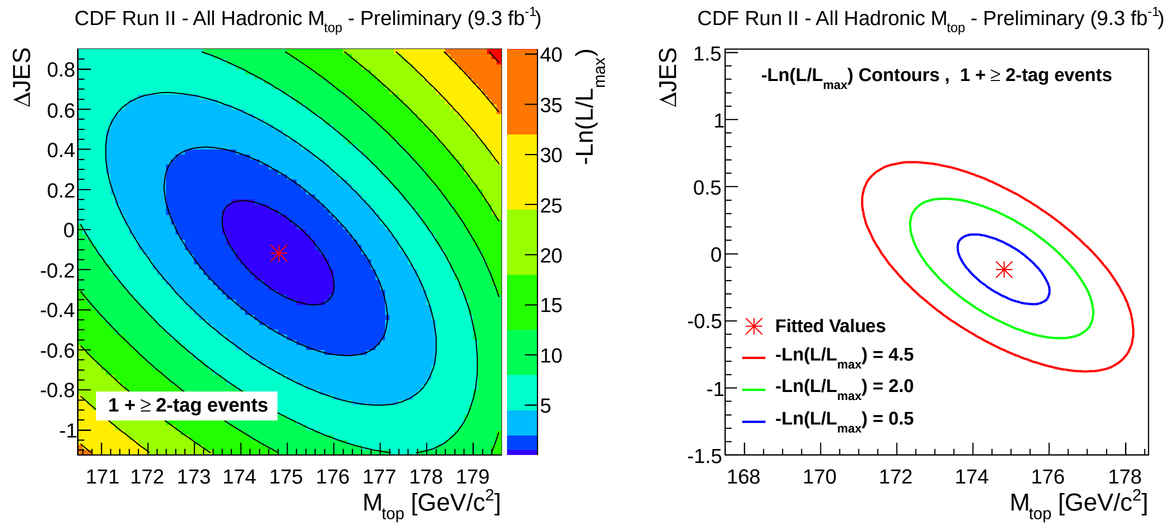

FIG. 11: Measured likelihood as a function of the $\mathrm{M_{top}}$ and ΔJES parameters (left) and contours corresponding to variations of the same parameters of one, two and three standard deviations as given by MINOS (right). The fitted central values, corresponding to the maximum likelihood (or minimum $-\ln\mathcal{L}$), are also shown.

These values have to be calibrated and the uncertainties have also to be corrected by multiplicative factors 1.024 and 1.004 for $\mathrm{M_{top}}$ and ΔJES respectively, as mentioned in section VIII B, so that we finally obtain :

$$\begin{aligned} \mathrm{M}_{\mathrm{top}}^{corr} &= 175.07 \pm 1.58\,(stat + Fit\,syst)\,\mathrm{GeV}/c^2 \\ \Delta\mathrm{JES}^{corr} &= -0.282 \pm 0.331\,(stat + Fit\,syst) \end{aligned}$$

The different contributions to the uncertainty can be isolated and the results written as :

$$\begin{aligned} \mathrm{M}_{\mathrm{top}} &= 175.07 \pm 1.19\,(stat) \pm 0.97\,(\mathrm{JES}) \pm 0.41\,(n_s,\, n_b,\, \mathcal{A}_b)\ \mathrm{GeV}/c^2 \\ \Delta\mathrm{JES} &= -0.282 \pm 0.255\,(stat) \pm 0.207\,(\mathrm{M_{top}}) \pm 0.040\,(n_s,\, n_b,\, \mathcal{A}_b) \end{aligned}$$

The whole set of parameters, as measured in the data by the likelihood fit, is summarized in Table III together with the corrected values.

Summarizing, including the systematic uncertainties, the measured values for the top quark mass and the jet energy scale are :

$$\begin{aligned} \mathrm{M}_{\mathrm{top}} &= 175.07\ \pm 1.58\,(stat + Fit\,syst) \pm 1.15\,(syst)\,\mathrm{GeV}/c^2 \\ \Delta\mathrm{JES} &= -0.28\ \pm 0.33\,(stat + Fit\,syst) \pm 0.44\,(syst) \end{aligned}$$

or, dividing completely the statistical and systematic contibutions

$$\begin{aligned} \mathrm{M}_{\mathrm{top}} &= 175.07\ \pm 1.19\,(stat)\ \pm 1.56\,(syst)\,\mathrm{GeV}/c^2 \\ \Delta\mathrm{JES} &= -0.28\ \pm 0.26\,(stat)\ \pm 0.49\,(syst) \end{aligned}$$

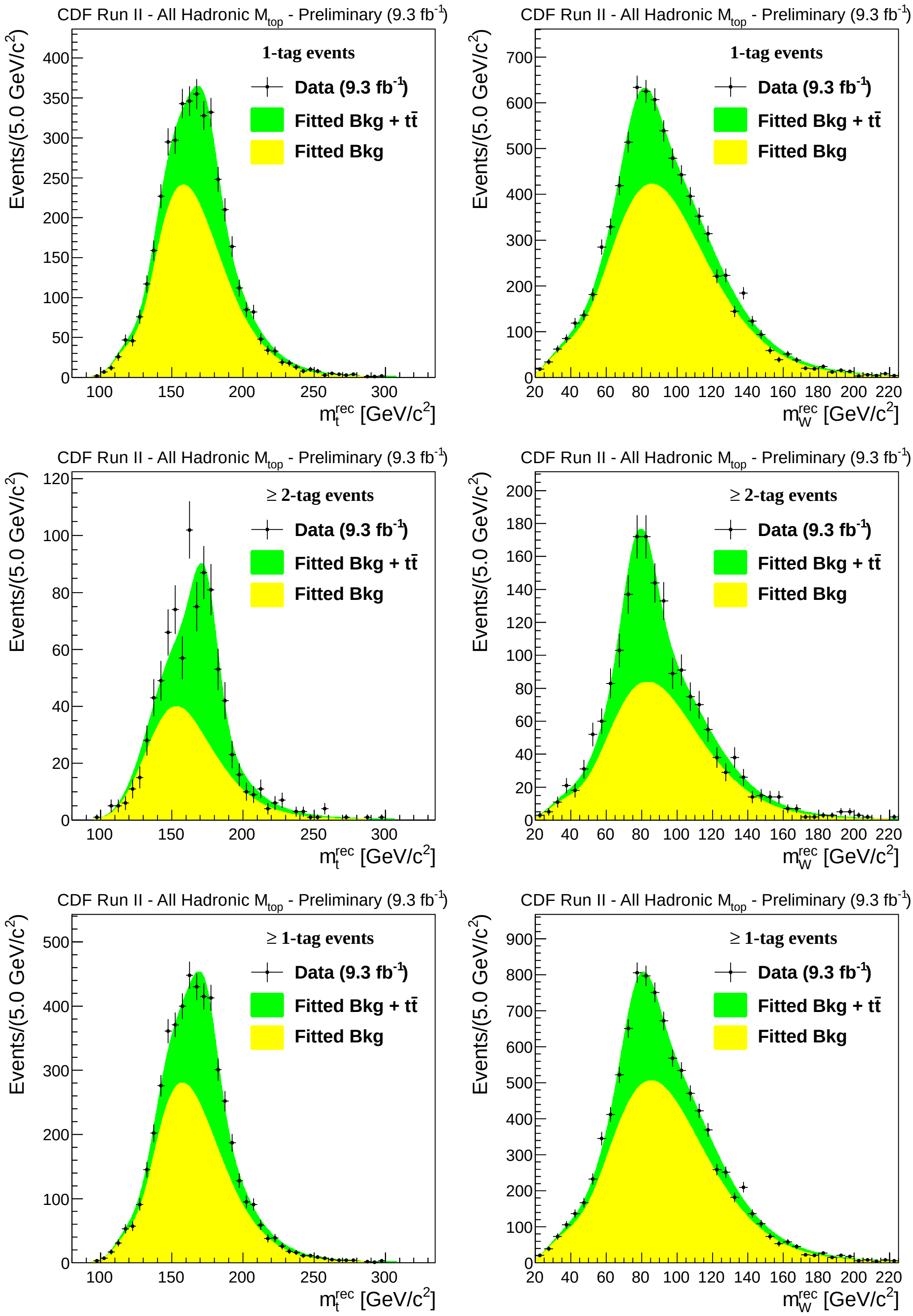

FIG. 12: Distributions of m_t^{rec} (left plots) and m_W^{rec} (right plots) as obtained in the data (black points) are compared to the probability density functions from signal and background corresponding, both in shape and normalization, to the likelihood fit parameters measured in the data. The upper and middle plots show distributions for the 1-tag and $\geq$ 2-tags samples respectively, while the lower plots are their sum.

CDF Run II - All Hadronic M_{top} - Preliminary ($9.3\,\mathrm{fb}^{-1}$)

Variable	Fitted value	Calibrated value
M_{top}	174.82 ± 1.25	175.07 ± 1.58
ΔJES	-0.118 ± 0.262	-0.282 ± 0.331
$n_s^{1\,tag}$ (JES-sample)	1862 ± 170	–
$n_b^{1\,tag}$ (JES-sample)	6028 ± 182	–
$\mathcal{A}_b^{1\,tag}$	0.478 ± 0.009	–
$n_s^{\geq 2\,tags}$ (JES-sample)	645 ± 58	–
$n_b^{\geq 2\,tags}$ (JES-sample)	1113 ± 62	–
$\mathcal{A}_b^{\geq 2\,tags}$	0.430 ± 0.021	–
$n_s^{1\,tag}$ (M_{top}-sample)	1244 ± 114	–
$n_b^{1\,tag}$ (M_{top}-sample)	2881 ± 87	–
$n_s^{\geq 2\,tags}$ (M_{top}-sample)	420 ± 38	–
$n_b^{\geq 2\,tags}$ (M_{top}-sample)	479 ± 27	–

TABLE III: The values of free parameters and their uncertainties as fitted by MINUIT in the data by the likelihood fit, and their values after the calibration. The numbers of events in the M_{top}-sample are derived from the corresponding values in the JES-sample and the acceptances $\mathcal{A}_s$ and $\mathcal{A}_b$.

The plots in Fig. 12 show the m_t^{rec} and m_W^{rec} distributions for the data compared to the probability density functions corresponding to the fitted values of M_{top} and ΔJES, In all these plots the signal and background contributions are normalized to the respective number of events as fitted in the data.

The plots in Fig. 13 compare the observed calibrated uncertainties, to the expected distribution from default pseudo-experiments using as input mass $M_{top} = 175.5\,\mathrm{GeV}/c^2$ and $\Delta\mathrm{JES} = -0.5$, i.e. the available templates with input top quark mass and ΔJES as close as possible to the values measured in the data. We find that the probability of achieving a better sensitivity is 80% for M_{top} and 65% for ΔJES.

XI. CONCLUSIONS

We described in this note the Template Method technique with *in situ* calibration used to measure the top quark mass on the latest available data sample, corresponding to an integrated luminosity of $9.3\,\mathrm{fb}^{-1}$. The method has been studied and calibrated through thousands of pseudo-experiments and the systematic uncertainties estimated by the same procedure. We then applied the technique to the data to measure a top quark mass of $175.07\ \pm 1.19\,(stat)\ \pm 1.56\,(syst)\,\mathrm{GeV}/c^2$.

Acknowledgments

We thank the Fermilab staff and the technical staffs of the participating institutions for their vital contributions. This work was supported by the U.S. Department of Energy and National Science Foundation; the Italian Istituto Nazionale di Fisica Nucleare; the Ministry of Education, Culture, Sports, Science and Technology of Japan; the Natural Sciences and Engineering Research Council of Canada; the National Science Council of the Republic of China; the Swiss National Science Foundation; the A.P. Sloan Foundation; the Bundesministerium für Bildung und Forschung, Germany; the Korean World Class University Program, the National Research Foundation of Korea; the Science and Technology Facilities Council and the Royal Society, United Kingdom; the Russian Foundation for Basic

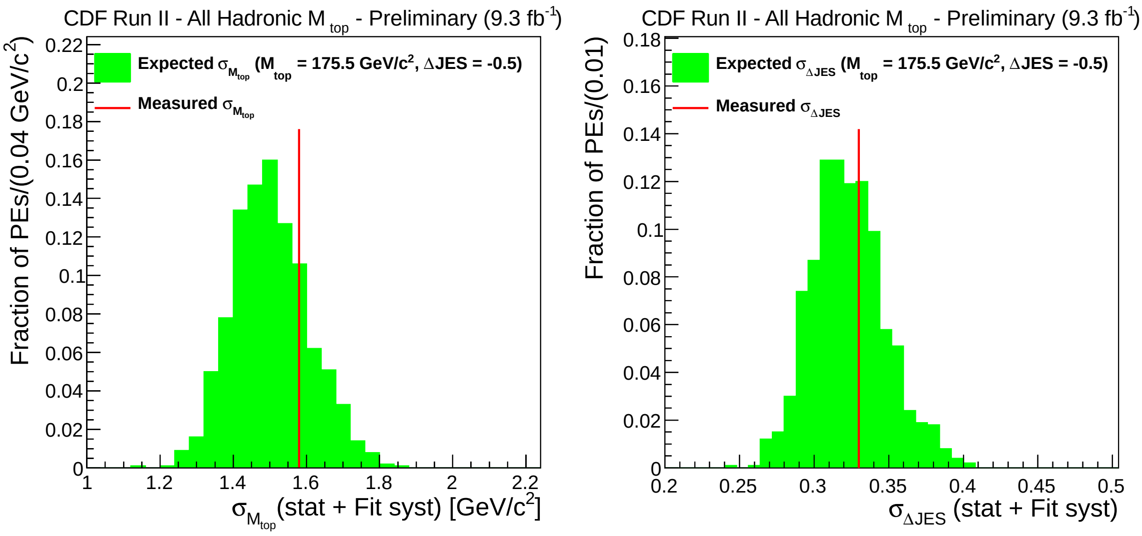

FIG. 13: Uncertainties on the top quark mass (left) and the jet energy scale displacement (right) as measured in default PEs performed at $M_{\rm top}^{in} = 175.5\,{\rm GeV}/c^2$ and JES $= -0.5$, i.e. using the available set of PEs with input top quark mass and ΔJES as close as possible to the values measured in the data. The red lines indicate the uncertainties obtained in the data.

Research; the Ministerio de Ciencia e Innovación, and Programa Consolider-Ingenio 2010, Spain; the Slovak R&D Agency; the Academy of Finland; the Australian Research Council (ARC); and the EU community Marie Curie Fellowship Contract No. 302103.

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[13] The ≥ 2-tags sample actually consists of events with 2 or 3 tagged jets. When 3 tags are present, the 3 different possible assignments of two out of three jets to b quarks are also tested, with the remaining tagged jet considered as a light quark
[14] Given the large number of pseudo-experiments, fluctuations due to the PEs statistic are negligible
[15] The pull of a corrected variable X^{corr} is defined as $\frac{X^{corr}-X^{in}}{\sigma^{corr}_{\rm M_{top}}}$, where X^{in} is the known input value of X
[16] The mean number of background events, n_b^{in}, is taken as the difference between the number of events observed in the data and n_s^{in}